FHDW b.i.b.
FACHHOCHSCHULE DER WIRTSCHAFT
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Lösung zu Folgen, Reihen und Grenzwerte

1. a) Geometrische Folge mit $a_1 = \frac{2}{3}$ und $q = \frac{2}{3}$. b) $\frac{64}{729}$

c) $s_5 = \frac{2}{3} \cdot \frac{1-\left(\frac{2}{3}\right)^5}{1-\frac{2}{3}} = \frac{2}{3} \cdot \frac{1-\frac{32}{243}}{\frac{1}{3}} = \frac{2}{3} \cdot 3 \cdot \frac{211}{243} = \frac{422}{243}$

2. Allgemeine Formel zur Lösung des Problems:

$$a_n = a_1 + (n-1) \cdot d \quad \text{mit}$$

n = Anzahl der Jahre, a_1 = Anfangswert, d = jährlicher Zuwachs,

$$s_n = \frac{a_1 + a_n}{2} \cdot n \quad 4 \cdot \frac{1-\left(\frac{1}{2}\right)^5}{1-\frac{1}{2}} = 4 \cdot 2 \cdot \left(1 - \frac{1}{64}\right) = 8 \cdot \frac{63}{64} = \frac{63}{8}$$

a) Mit n = 7, a_1 = 200 und d = 50 folgt:

$a_7 = 200 + 6 \cdot 50 = 500$, $s_7 = \frac{200+500}{2} \cdot 7 = 2.450$

b) $a_{10} = 200 + 9 \cdot 50 = 650$.

3.

i	1	2	3	4	5
x_i	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$
y_i	3	-3	7	-7	11
z_i	1	2	2	4	8
v_i	5	$-\frac{1}{2}$	$\frac{28}{3}$	$-\frac{11}{4}$	$\frac{96}{5}$

4. a) $\frac{3+60}{2} \cdot 20 = 630$ b) $\frac{7+43}{2} \cdot 10 = 250$

5. a) $8 \cdot \frac{2^5 - 1}{2-1} = 8 \cdot 31 = 248$ b) $4 \cdot \frac{1-\left(\frac{1}{2}\right)^6}{1-\frac{1}{2}} = 4 \cdot 2 \cdot \left(1 - \frac{1}{64}\right) = 8 \cdot \frac{63}{64} = \frac{63}{8}$

9. a) $8 \cdot \frac{2^5-1}{2-1} = 8 \cdot 31 = 248$;

b) $4 \cdot \dfrac{1-\left(\frac{1}{2}\right)^6}{1-\frac{1}{2}} = 8 \cdot \left(1 - \frac{1}{64}\right) = 8 \cdot \frac{63}{64} = \frac{63}{8} = 7\frac{7}{8}$.

8.2 Grenzwerte von Folgen

1. a) $\lim\limits_{n\to\infty} \dfrac{2n+1}{n} = \lim\limits_{n\to\infty}\left(2+\dfrac{1}{n}\right) = 2 + \lim\limits_{n\to\infty}\dfrac{1}{n} = 2$;

b) Die Folge ist divergent.

2. a) $\lim\limits_{n\to\infty} \dfrac{7n^2-4n+8}{5n^2-6n} = \lim\limits_{n\to\infty} \dfrac{7-\frac{4}{n}+\frac{8}{n^2}}{5-\frac{6}{n}} = \frac{7}{5} = 1{,}4$;

b) $\lim\limits_{n\to\infty}\left(3-\dfrac{3}{n}\right) = 3 - \lim\limits_{n\to\infty}\dfrac{3}{n} = 3-0 = 3$.

3. Richtig: **b)**, **d)** .

4. $\frac{1}{6} : \frac{1}{2} = \frac{1}{3}$.

8.3 Grenzwerte von Reihen

1. Anfangsglied: $a = \frac{1}{3} \cdot 5 \cdot \frac{1}{9} = \frac{5}{27}$; $q = \frac{1}{3}$; $S = \frac{5}{27} \cdot \dfrac{1}{1-\frac{1}{3}} = \frac{5}{18}$.

2. $\sum\limits_{i=2}^{\infty}\left(\frac{1}{4}\right)^{i-1} = \dfrac{\frac{1}{4}}{1-\frac{1}{4}} = \dfrac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$.

3. $\sum\limits_{n=0}^{\infty}(1-a)^n = \dfrac{1}{1-(1-a)} = \dfrac{1}{a}$; $0 < a < 1$.

4. a) divergent ; b) $\dfrac{\frac{1}{4}}{1-\frac{1}{2}} = \frac{1}{2}$.

5. Richtig: **b)** .

6. $\sum\limits_{i=1}^{\infty}\left[\left(\frac{1}{6}\right)^{i-1} + \left(\frac{1}{11}\right)\right] = \sum\limits_{i=1}^{\infty}\left(\frac{1}{6}\right)^{i-1} + \sum\limits_{i=1}^{\infty}\left(\frac{1}{11}\right)^{i}$

$= \dfrac{1}{1-\frac{1}{6}} + \dfrac{1}{11} \cdot \dfrac{1}{1-\frac{1}{11}} = \frac{6}{5} + \frac{1}{10} = \frac{13}{10} = 1{,}3$.

7. $S = \sum_{i=-2}^{\infty} 3\left(\frac{1}{3}\right)^{i-2} + \sum_{i=-2}^{\infty} 6\left(\frac{1}{4}\right)^{i+1}$; $\quad q_1 = \frac{1}{3} \quad q_2 = \frac{1}{4}$

$a_1 = 3\left(\frac{1}{3}\right)^{-4} = 3 \cdot 3^4 = 243$; $\quad a_2 = 6\left(\frac{1}{4}\right)^{-1} = 6 \cdot 4 = 24$;

$$S = \frac{243}{1-\frac{1}{3}} + \frac{24}{1-\frac{1}{4}} = \frac{729}{2} + 32 = 364{,}5 + 32 = 396{,}5 \ .$$

8. $\frac{1}{\frac{2}{3}} + \frac{16}{\frac{3}{4}} = \frac{3}{2} + \frac{64}{3} = \frac{137}{6} = 22\frac{5}{6}$.

8.4.1 a) $a = 5, q = \frac{1}{3}, S = \frac{15}{2}$; b) $a = 6, q = \frac{1}{4}, S = 8$.

8.4.2 a) $a = 12 \cdot \frac{1}{4} = 3, q = \frac{1}{4}, S = 4$;

b) $a = (\frac{1}{4})^{-1} = 4, q = \frac{1}{4}, S = \frac{16}{3}$;

c) $a = 20 \cdot (\frac{1}{5})^2 = \frac{4}{5}, q = \frac{1}{5}, S = 1$;

d) $a = (\frac{1}{2})^{-1} = 2, q = \frac{1}{2}, S = 4$;

e) $a = (\frac{1}{3})^{-4} = 81, q = \frac{1}{3}, S = 121{,}5$.

8.4.3
$$\sum_{n=0}^{\infty}(-1)^n\frac{1}{2^n} = \sum_{k=0}^{\infty}\left(\frac{1}{2^{2k}} - \frac{1}{2^{2k+1}}\right) = \sum_{k=0}^{\infty}\left(\frac{1}{4^k} - \frac{1}{2\cdot 4^k}\right)$$
$$= \sum_{k=0}^{\infty}\frac{1}{4^k}\left(1-\frac{1}{2}\right) = \frac{1}{2}\sum_{k=0}^{\infty}\frac{1}{4^k} = \frac{1}{2}\cdot\frac{1}{1-\frac{1}{4}} = \frac{2}{3}.$$